

Kindergarten Mathematics

Scope and Sequence

Quarter 1

Unit of Study 1.1: Building, Drawing, and Analyzing Shapes: Basic 2-D shapes; matching, identifying, sorting, building, describing shapes in their environment (1st full week of school/5 days)

Standards for Mathematical Content

Geometry K.G

Analyze, compare, create, and compose shapes.

K.G.2 Correctly name shapes regardless of their orientation or overall size

K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

Match two dimensional shapes, name shapes, distinguish from non-examples of the categories. Describe physical world from geometric perspectives---shape, orientation, and spatial relations. Sort shapes according to categories, explain names of classification, attend to and describe certain features. Classify in different ways by attributes

Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Unit of Study 1.2: Developing an understanding of numbers 0-5 by writing and representing numbers. Develop rapid visual and kinesthetic recognition of number to 5 on fingers (10 days)

Standards for Mathematical Content

Counting and Cardinality K.CC

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

Say counting words first then use these numbers to count objects or to tell the number of objects. Pair each word with one object. Facilitate by pointing to objects or moving them. Count object in a line, later count in arrangements. Lastly count out a given number of objects.

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

Develop rapid visual and kinesthetic recognition of numbers to 5 on their fingers. Using fingers not a concern unless it remains at the first level of direct modeling in later grades.

K.CC.5 Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Say counting words first then use these numbers to count objects or to tell the number of objects. Pair each word with one object. Facilitate by pointing to objects or moving them. Count object in a line, later count in arrangements. Lastly count out a given number of objects

Evidence Statements:

- i) Tasks may have context.***
- ii) Tasks should include a range of counting exercises to answer “how many” objects in different arrangements progressing to the more difficult action of counting out a given number of objects.***
- iii) Interviews (individual or small group) should target students’ abilities to meet this evidence statement.***

Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

2 Reason abstractly and quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Unit of Study 1.3: Developing Understanding of Numbers 0–10 and writing and representing numbers 0-10 (10 days)

Standards for Mathematical Content

Counting and Cardinality K.CC

Know number names and the count sequence.

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell how many objects

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

- a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
- b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- c. Understand that each successive number name refers to a quantity that is one larger.

Count to tell the number of objects.

K.CC.5 Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Say counting words first then use these numbers to count objects or to tell the number of objects. Pair each word with one object. Facilitate by pointing to objects or moving them. Count object in a line, later count in arrangements. Lastly count out a given number of objects.

Evidence Statements

- i) **Tasks may have a context.**
- ii) **Tasks should include a range of counting exercises to answer “how many” objects in different arrangements progressing to the more difficult action of counting out a given number of objects.**
- iii) **Interviews (individual or small group) should target students’ abilities to meet evidence statement.**

Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Unit of Study 1.4: Understanding and describing shapes and space, spatial relationships and structuring, and orientation (i.e. above/below, next to, beside, etc.) (10 days)

Standards for Mathematical Content

Geometry K.G

K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind, and next to*

Describe physical world from geometric perspectives - shape, orientation and spatial relations

Standards for mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Unit of Study 2.1: Classification and Sorting of objects into categories; more or less up to 10 (5 days)

Standards for Mathematical Content

Counting and Cardinality K.CC

Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.¹

¹ Include groups with up to ten objects.

Classify objects into categories, initially specified by the teacher eventually solicited from the students. Ask students to sort, count, say whether there are more or less.

Recognize the cardinalities of small groups then develops into conceptual subitizing (recognizing that a collection is composed of two sub collections (seeing two sets of 2 and saying 4).

Geometry KG

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.2 Correctly name shapes regardless of their orientations or overall size.

Measurement and Data

Classify objects and count the number of objects in each category.

K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.³

³Limit category counts to be less than or equal to 10.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students

might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, —Does this make sense?|| They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Unit of Study 2.2: Developing an Understanding of Numbers 11–20; Writing and Representing numbers 11-20 (10 days)

Standards for Mathematical Content

Number and Operations in Base Ten K.NBT

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.¹

¹Pay special attention to 10. View whole numbers 11-19 as ten ones and some more numbers. Decompose 10 into pairs (9+1, 7+3 etc.) and find the number that makes 10 when added to a given number.

Evidence Statements:

- i) Tasks should focus on the understanding of numbers from 11 to 19 as composed of ten “ones” and some additional number of “ones”.**
- ii) Tasks should require students to record their thinking with a drawing or equation.**
- iii) Interviews (individual or small group) should target this understanding of composing and decomposing the teen numbers into ten “ones” and some additional number of “ones”.**

Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions as single objects or as being composed of several objects. For example they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real number x and y .

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y-2)/(x-1)=3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Unit of Study 2.3: Counting and Writing to 20; Counting Forward to 20 From Any Number (10 days)

Standards for Mathematical Content

Counting and Cardinality K.CC

Know number names and the count sequence.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).¹

¹ Counting forward is a prerequisite to counting on. It is beginning with a given number within the known sequence; finally understand that each successive number name refers to a quantity that is one larger.

Counting on—understand the last number name said in counting tells the number of objects counted; then understand the number of objects is the same regardless of their arrangement or the order in which they are counted.

K.CC.3 Write number from 0 to 20. Represent a number of objects with a written numeral 0 to 20 (with 0 representing a count of no objects).

Evidence Statements:

- i) Tasks may have a context**
- ii) Tasks should include a range of counting exercises to answer “how many” objects in different arrangements progressing to the more difficult action of counting out a given number of objects.**
- iii) Interviews (individual or small group) should target students’ abilities to meet this evidence statement.**

Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions as single objects or as being composed of several objects. For example they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real number x and y .

Unit of Study 2.4: Addition within 5 with objects, fingers, and math drawings. Utilizing multiple positions of equations. (10 days)

Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings², sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

2 Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Focus on quantities and their relationships. Can be acted out or presented with pictures or words. Drawings facilitate reflection and discussion.

Equations with one number on the left and an operation on the right ($5=2+3$), allow understanding equations as showing various ways that the quantities on both sides have same value.

K.OA.5 Fluently add and subtract within 5.

Evidence Statement:

- i) Tasks should provide students opportunities to demonstrate fluency for addition and subtraction within 5 and to apply different solution methods.
- ii) Interviews (individually or small group) should target students' abilities to meet this evidence statement.

Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions as single objects or as being composed of several objects. For example they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real number x and y .

Unit of Study 3.1: Operations and Algebraic Thinking: Introduce subtraction within 5 with objects, their fingers and math drawing (10 days)

Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.5 Fluently add and subtract within 5.

Students solve add/subtraction equations for numbers within 5. ($2+1=?$, $3-1=?$) Experience with decomposition of numbers and Add To and Take From situations enables student to begin to fluently add and subtract within 5.

Evidence Statement:

- i) Tasks should provide students opportunities to demonstrate fluency for addition and subtraction within 5 and to apply different solution methods.**
- ii) Interviews (individual or small group) should target students' abilities to meet this evidence statement.**

Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

Unit of Study 3.2: Operations and Algebraic Thinking: Mixing addition and subtraction equations with I-5 using objects, fingers, math drawings and role play (10 days)

Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings², sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

² Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Students begin Put Together/Take Apart situations acting out with objects and later children begin to move to conceptual mental acts (seeing children are seeing boys and girls, or seeing red and green apples or all the apples). Allows to explore various compositions (partners) that make each number. Find patterns in the compositions of a given number and summarize these patterns.

Evidence Statements:

- i) Tasks should include the following problem situations: “Add To” and “Take From” – Result Unknown Problems, and “Put Together/Take Apart” – Total Unknown and Both Addends Unknown Problems (for more information see CCSS Table 1, p. 88 and OA Progression, p. 9)
- ii) Interviews (individual or small group) are used to assess mastery of different problem types.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Unit of Study 3.3: Measurement: Compare length (align end points) and weight. Describe and compare measurable attributes. Represent measurement data on picture/bar graph. (10 days)

Standards for Mathematical Content

Measurement and Data K.MD

Describe and compare measurable attributes.

K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Describe and compare measurable attributes- conversations about how students compare help them to learn to discriminate and name these measurable attributes and student learn to distinguish, label, and describe several measurable attributes of a single object.

Teachers listen and extend conversation about things that are big, tall, long, small, high, and name, discuss and demonstrate with gestures the attribute.

Length – conversation occur in comparing situations. These align one endpoint. Help students resolve and coordinate perceptual and conceptual information.

Necessity of aligning endpoint should be discussed explicitly. Then reintroduced in many situations that call for comparison.

Learn to compare lengths of two objects using a third object, order lengths, and connect number to length.

Area and volume are not instructional focus for kindergarten.

Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

Unit of Study 3.4: Geometry: Basic 3-D shapes; matching, identifying, sorting, describing. Describe shapes in their own environment. (5 days)

Standards for Mathematical Content

Geometry K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.3 Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

Analyze, compare, create, and compose shapes.

K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).

K.G.6 Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?”

Perceive a variety of shapes in their environment, describe shapes in own words recognize and informally name 3 dimensional shapes e.g., balls, boxes, cans; in context of solving problems in block building drawing pictures and simple maps.

Name and describe 3 dimensional shapes with math vocabulary (sphere, cube, cylinder, and cone. ID faces of 3D shapes as 2D geometric figures. ID as 2D (flat) or 3D (solid).

Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

Unit of Study 3.5: Counting and Cardinality: Identify whether the number of objects in one group is greater than, less than, or equal to objects in another group (0-10) (10 days)

Standards for Mathematical Content

Counting and Cardinality K.CC

Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.¹

¹ Include groups with up to ten objects.

Comparing numbers 0-10 in two groups with more or fewer than or the same amount. First learn to match the two groups and look for extras, then count each group to decide which number is greater.

Counting and Cardinality K.CC

Compare numbers.

K.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, —Does this make sense? They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Unit of Study 4.1: Operations and Algebraic Thinking: Decompose numbers less than or equal to 10 into pairs. Developing equations in more than one way. (10 days)

Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).

Working within 10-use the level 1 methods as did with 0-5. Patterns such as “adding one is just the next counting word” and “adding zero gives the same number” useful from numbers 1 through 9. $(5+n)$ and patterns in Level 2 and 3 methods, relationships should be explained explicitly as your progress, mental images become important in problem representation.

Evidence Statement:

- i) Tasks may have a context.**
- ii) Tasks should include a range of activities tht focus on decomposing numbers less than or equal to 10 into pairs in more than one way.**
- iii) Tasks require students to record their thinking with a drawing or equation.**
- iv) Interviews (individual or small group) should target students’ abilities to meet this evidence statement.**

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

Evidence Statement:

- i) Tasks may have a context.**
- ii) Tasks should focus on students’ understanding of making 10 and representing their thinking.**
- iii) Interviews (individual or small group) should target students’ abilities to meet this evidence statement.**

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, —Does this make sense? They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Unit of Study 4.2: Adding and Subtracting up to 10 (10 days)

Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings², sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

2 Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Relationships in add/sub for larger numbers 6-9 show the relationships in various ways: things crossed out, numbers labeling parts or totals and letters or words labeling aspects of situation. Encourage students to explain their drawings and discuss similarities/differences in drawings.

Evidence Statements:

- i) Tasks should include the following problem situations: “Add To” and “Take From” – Result Unknown Problems, and “Put Together/Take Apart” – Total Unknown and Both Addends Unknown Problems (for more information see CCSS Table 1, p. 88 and OA Progression, p. 9)
- ii) Interviews (individual or small group) are used to assess mastery of different problem types.

K.OA.4 For any number 1 to 9, find the number that makes 10 when added to the given number, e.g. by using objects or drawings, and record the answer with a drawing or equation.

Evidence Statements:

- i) Tasks may have a context.
- ii) Tasks should focus on students’ understanding of making 10 and representing their thinking.
- iii) Interviews (individual or small groups) should target students’ abilities to meet this evidence statement.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain

correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.

Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, —Does this make sense? They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Unit of Study 4.3: Operations and Algebraic Thinking: Developing Foundations for Place Value, 11–19 (10 days)

Standards for Mathematical Content

Number and Operations in Base Ten K.NBT

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Composing and decomposing numbers from 11-19 into ten ones and some further ones, builds from all this work, vital first step toward understanding base-ten notation for numbers greater than 9.

Work with numbers 11-19 to gain foundation for place value. Use objects, math drawings and equations to describe, explore and explain how the “teen numbers” are ten ones and some more ones. Count out a given teen number of objects, e.g. 12. Special attention to 11, 12, 13 to understand them due to English language. E.g. layered place value cards help see the 0 “hiding” under the ones place and the 1 in the tens place really is 10 (ten ones).

Evidence Statements

- i) Tasks should focus on the understanding of numbers from 11 to 19 as composed of ten “ones” and some additional number of “ones”.***
- ii) Tasks should require students to record their thinking with a drawing or equation.***
- iii) Interview (individual or small group) should target this understanding of composing and decomposing the teen numbers into ten “ones” and some additional number of “ones”.***

Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y-2)/(x-1)=3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.